Modeling and Optimization Problems in Contact Centers (a biased overview)

Pierre L'Ecuyer

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Sponsored by Bell Canada

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- General overview and problem statements; importance of contact centers;
- Quantitative evaluation: queueing approximations vs simulation;
- Building realistic models;
- Optimization of staffing, scheduling, call routing, priorities, outbound call, etc.
- Simulation tools and improving simulation efficiency;

For my articles, Google "Pierre L'Ecuyer"

See also the references in the paper: Gans, Koole, Mandelbaum, Whitt, etc.

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K call types. Depends on required technical skill, language, importance, etc.
I agent types (or skill groups). Each has skills to handle certain call types.
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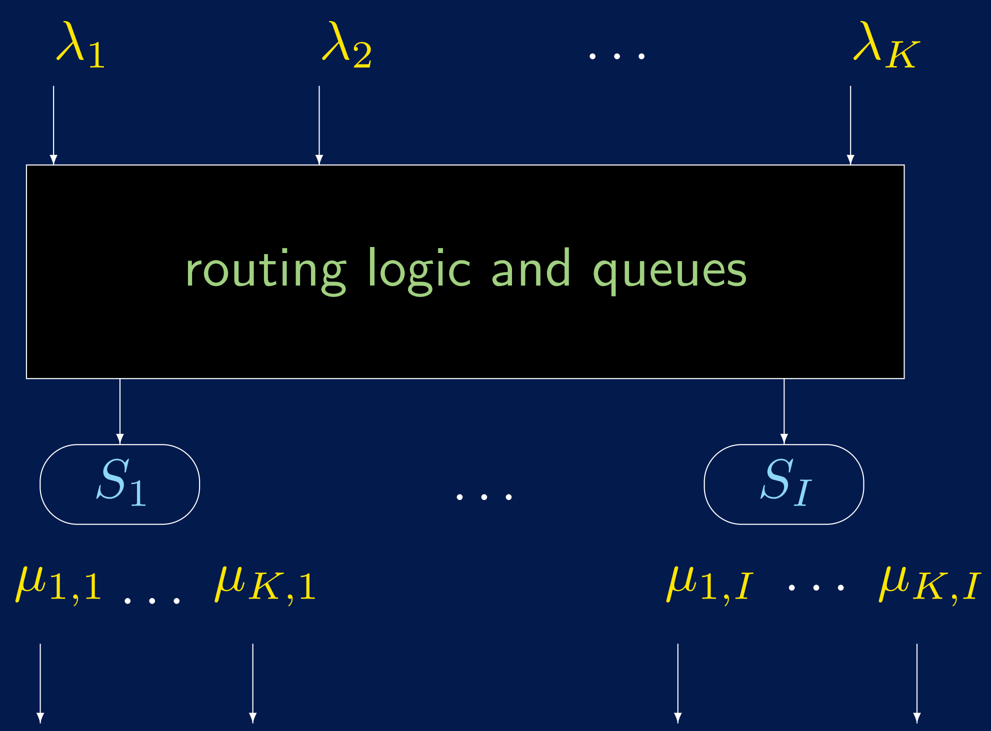

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For well-balanced systems, one or two skills per agent often gives a performance almost as good as all skills for all agents (e.g., Wallace and Whitt 2004).

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Occupation ratio of agents, per type and per period.
Should not exceed 90–92% (for fairness, stress, ...).

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Rules that control in real time the agent-to-call and call-to-agent assignments.
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Example: Weight the calls (e.g., waiting time so far, multiplied by an importance factor that depends on call type).
Next available agent picks call with largest weight among the types he can handle.

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Scheduling and rostering problem:
In practice, we do not have an infinite supply of each agent type!
For a given set of agents and a given set of admissible shifts, assign a shift to each agent, to meet the performance constraints at minimal cost.

Recourses against uncertainty

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Would need flexibility to change staffing levels on short notice.

Possibilities:
— move meetings and training sessions to low-volume periods;
— agents on standby at home;
— outsourcing part of the load (pass some of the risk, like with an insurance).

Blend systems: inbound and outbound calls

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Other types of recourses:

Controlling the arrival rate, e.g., by giving information on current waiting times?

Tell people to call again later.

Other ways?

Besides routing and scheduling:

Long-term strategic decisions

- Size and layout of center
- Call types, skill groups
- Types of work schedules
- Outsourcing decisions and contracts

Medium-term planning

- Hiring and training of new agents
- Training current agents

Building realistic models

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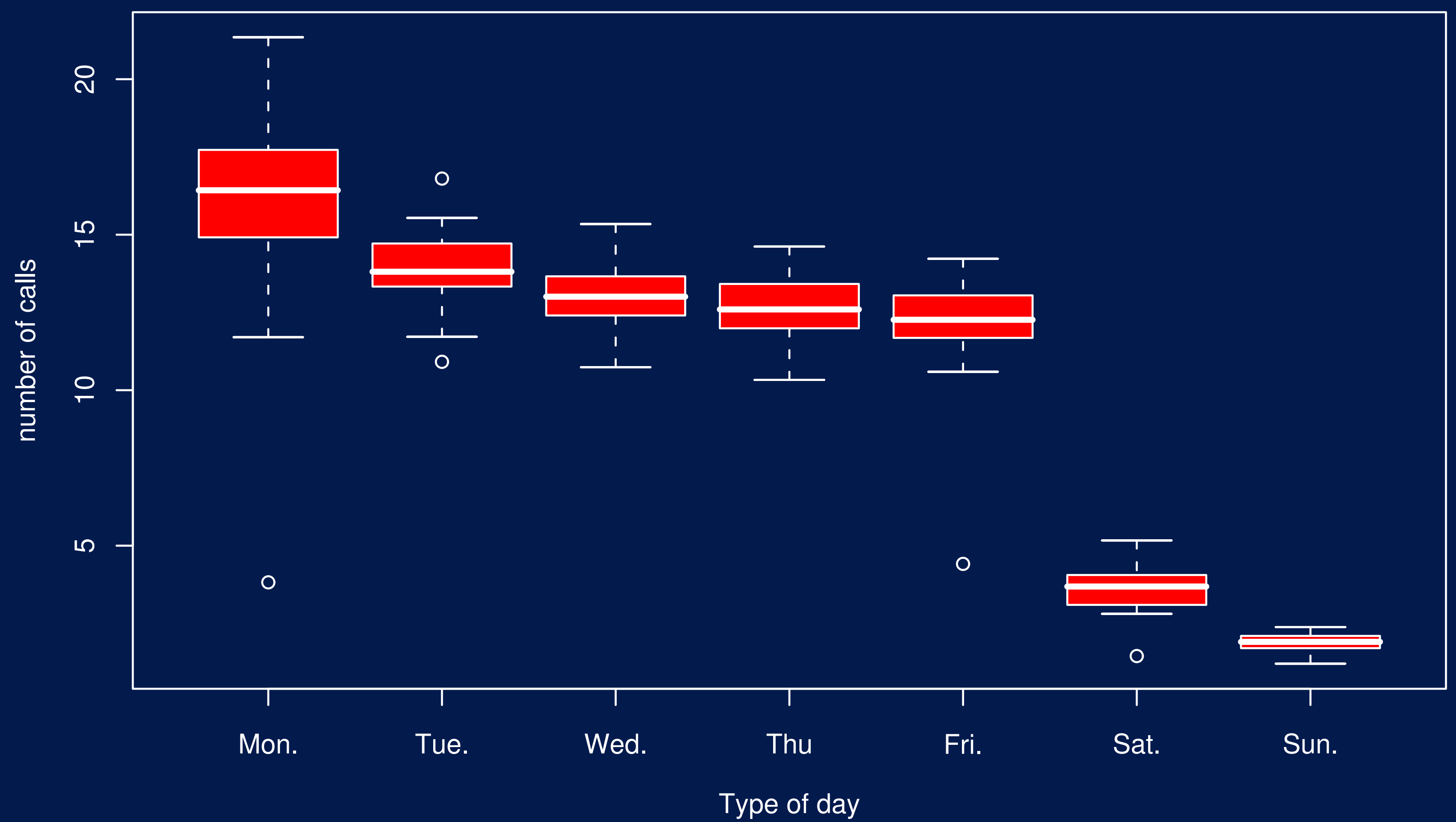
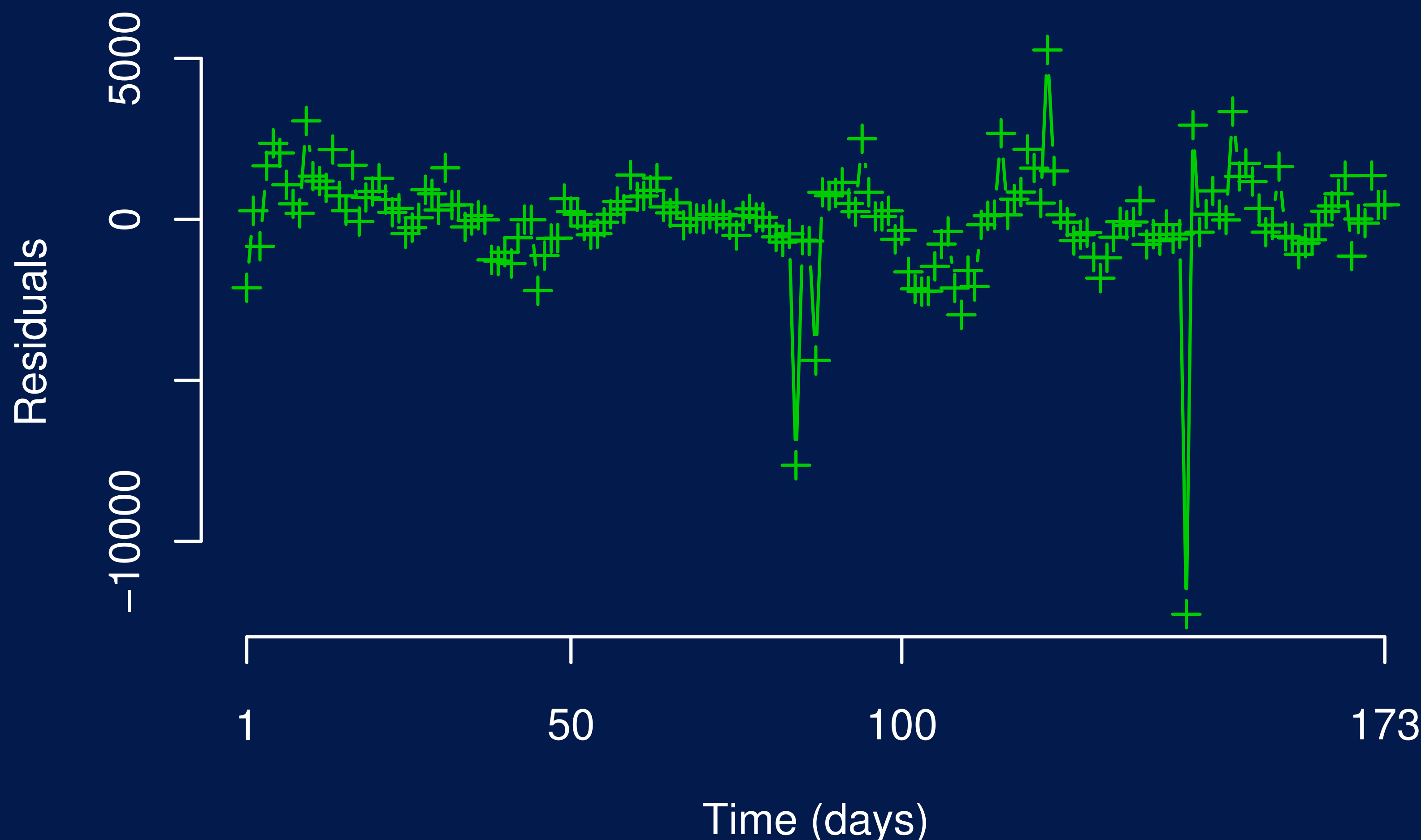
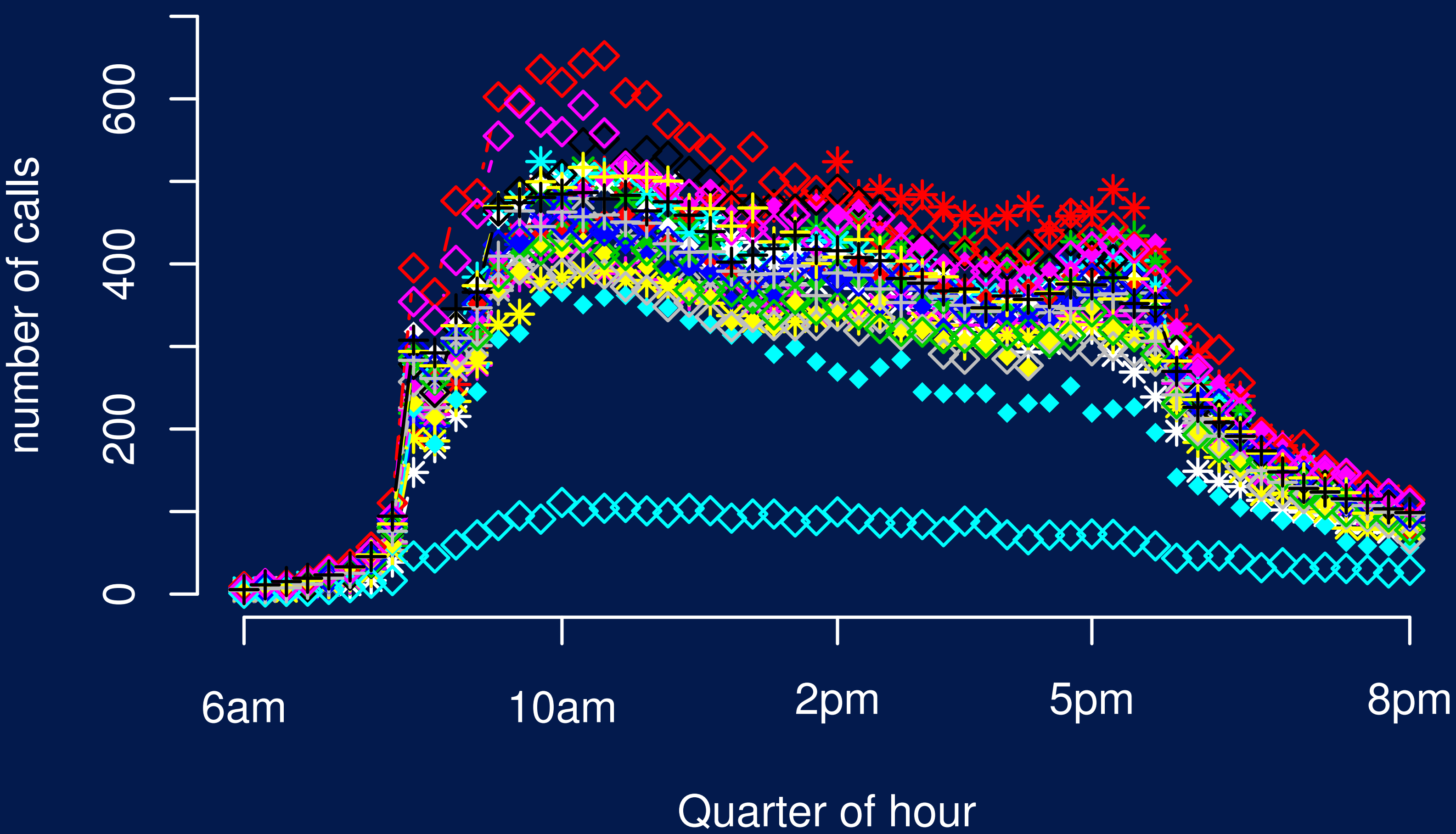
Reality: service times are not exponential; arrivals are not Poisson and stationary, breaks are not exactly as planned, agents are not available exactly as planned (e.g., they may sometimes disconnect themselves for a short period between two calls), etc.



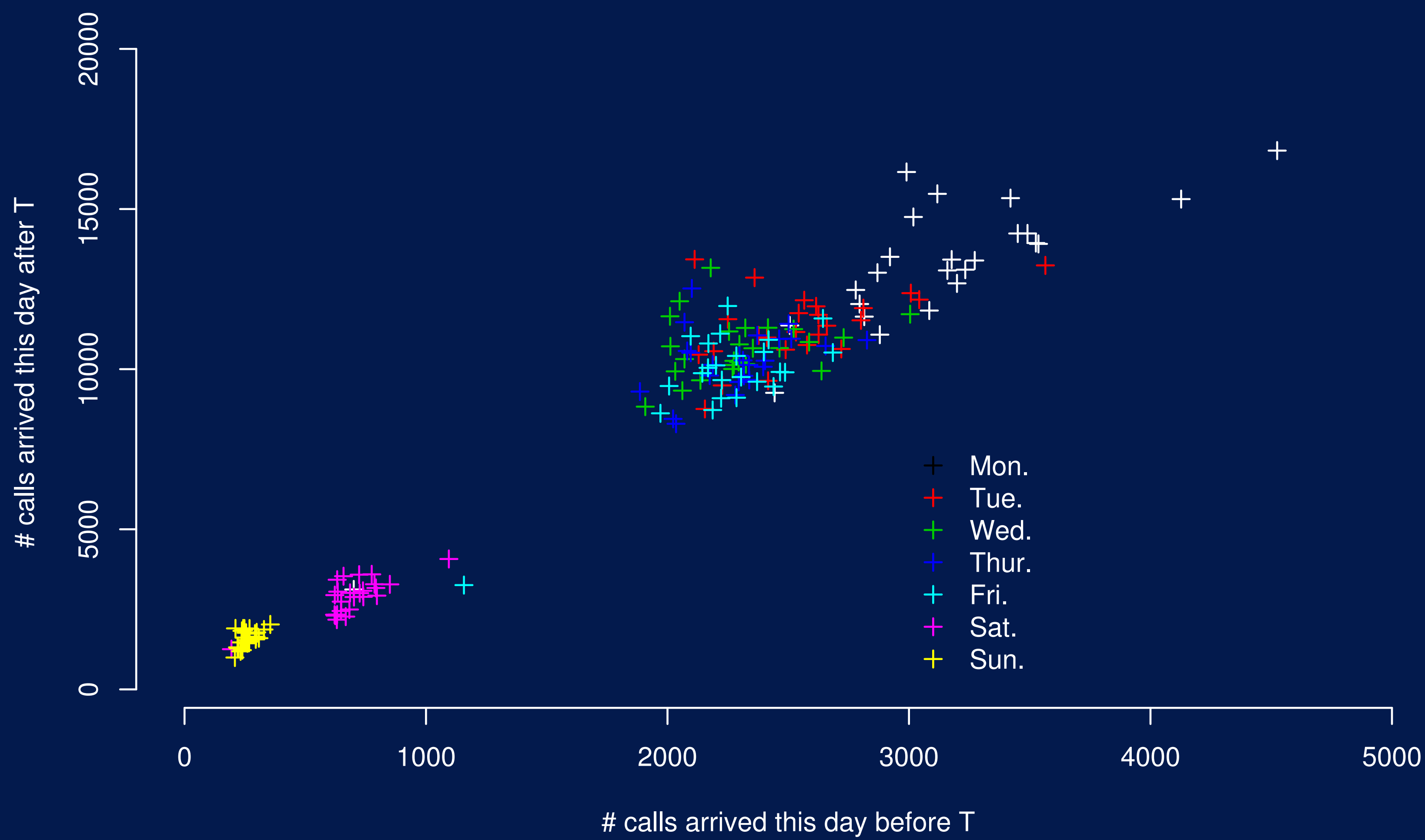
Arrival counts on different days



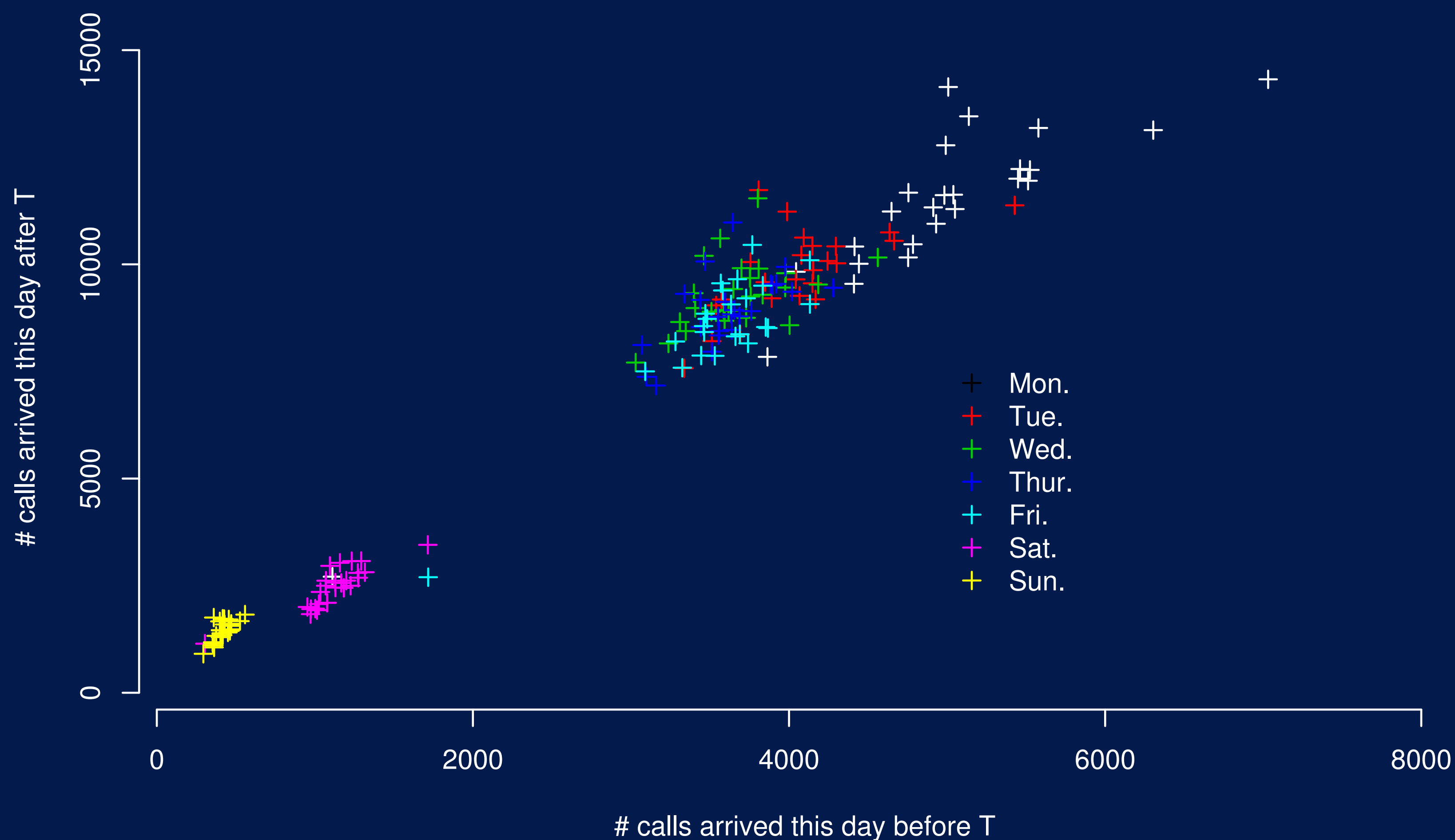
Correlations between successive days:
Daily residuals after removing day-effect, Box-Ljung test: p-value $= 7.5 \times 10^{-6}$



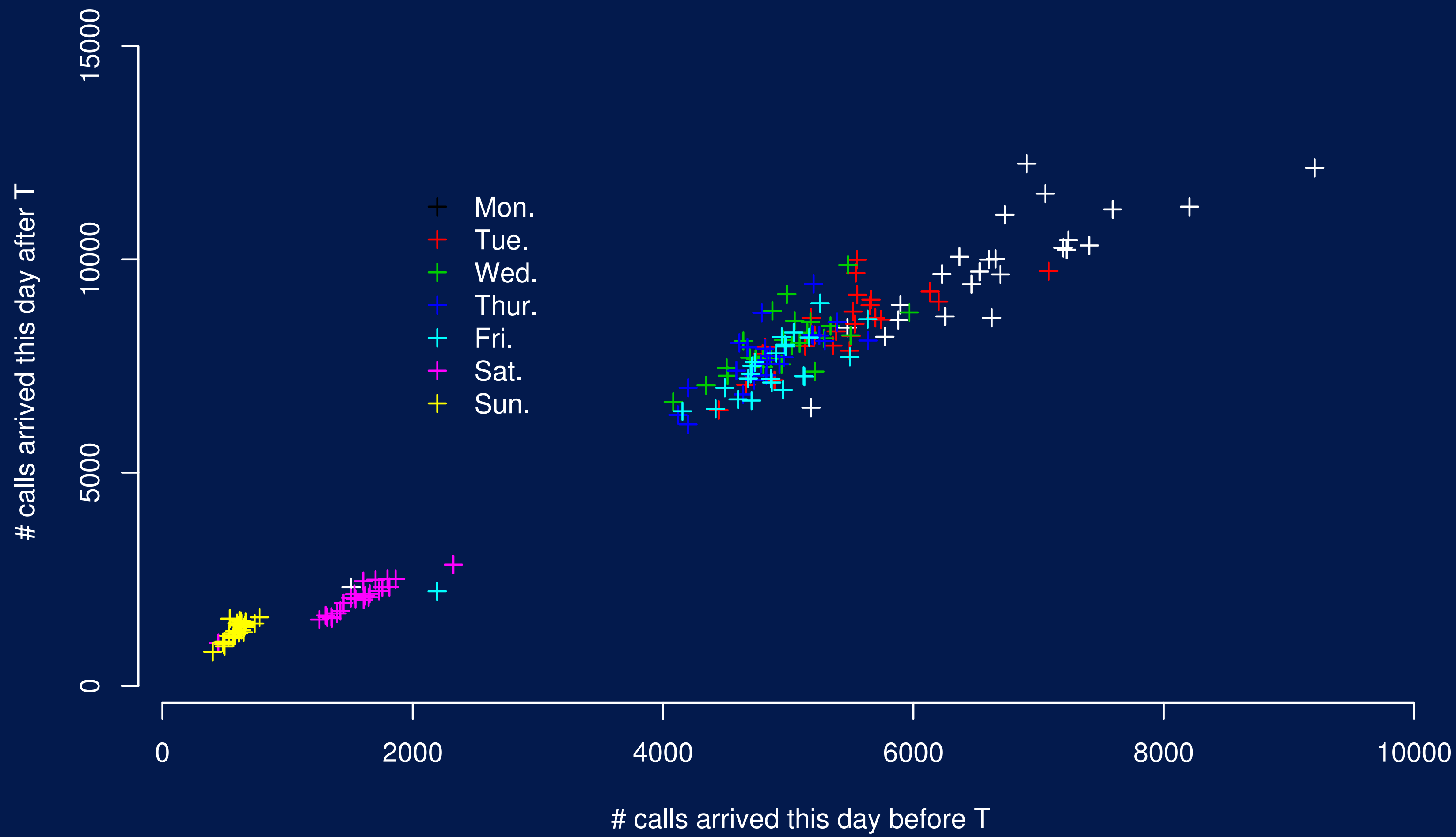
Monday, arrival counts by quarter hour



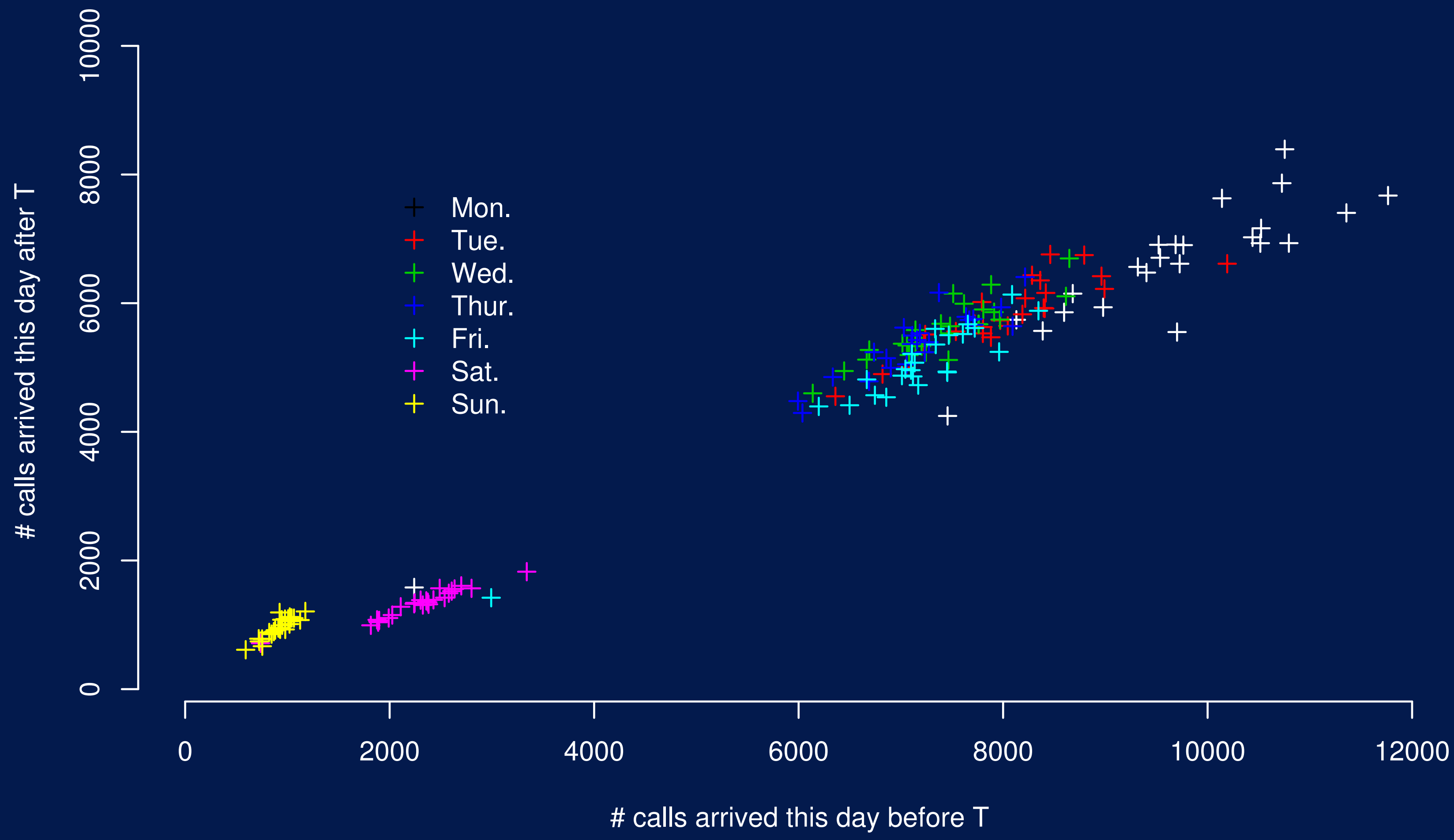
Scatter plot: (calls before T, calls after T), all days, $T = 10$ a.m.



Scatter plot, all days $T = 11$ a.m.



Scatter plot, all days $T = 12$ a.m.



Scatter plot, all days $T = 2$ p.m.

Arrival process modeling

Arrival processes can be nonstationary, doubly stochastic, dependent, etc.
Examples, for single type: nonstationary Poisson with random inflation factor each day; arrival rate $B\lambda(t)$ at time t, where $\mathbb{E}[B] = 1$.
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Service time distributions: lognormal often fits well. Sometimes loglogistic.
Patience time distributions;
Disconnections;
Etc.

Steady-state approximations of SL

Single skill: M/M/s (Erlang-C Analysis).

μ = service rate; λ = arrival rate; $r = \lambda/\mu$ = load
s = staffing (number of servers)
$\rho = \lambda/s\mu$ = utilization factor
W = steady-state delay in queue

Delay probability:

$$\mathbb{P}[W > 0] \; = \; \frac{\dfrac{r^s}{s!}\dfrac{s}{s-r}}{\dfrac{r^s}{s!}\dfrac{s}{s-r} + \displaystyle\sum_{i=0}^{s-1} \frac{r^i}{i!}}$$

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Service level:

$$\mathbb{P}[W > \tau] = \mathbb{P}[W > 0]\exp[(s\mu - \lambda)\tau]$$

(conditional waiting time is exponential with mean $1/(s\mu - \lambda)$.

Large system: square-root safety staffing.

Halfin-Whitt (1981) limit: sequence of M/M/s queues where μ is fixed, $s \to \infty$, and $(1-\rho)\sqrt{s} \to \beta$ for $0 < \beta < 1$.

Theorem. Under this regime,

$$\mathbf{P}(W > 0) \;\to\; \delta = \frac{1}{1 + \beta\Phi(\beta)/\phi(\beta)}$$

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Square-root formula: For load r and delay probability δ, staff

$$s = r + \beta\sqrt{r}$$

agents. Simpler than inverting M/M/s formula.

The safety staffing as a fraction of the load is $\beta/\sqrt{r} \to 0$.

Other asymptotic regimes.

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Quality and efficiency driven (QED) regime:
$\mathbb{P}[W > 0]$ is fixed in $(0, 1)$ (e.g., Halfin-Whitt).
This regime is appropriate for typical large call centers.

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In the limit when mean patience times $\to 0$, we have a loss system: Erlang-B).

Time-dependent arrival rate.

Pointwise Stationary (PS) approximation [Green and Kolesar 1991, Whitt 1991]:

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2. integrate result (performance) with respect to t.

Improvement: use arrival rate at t for performance at $t + \delta$, for some delay δ [e.g., Ingolfsson et al. 2000, Green and Kolesar 1991].

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Blocking probabilities per skill group and loss rates per call type are computed by solving a system of nonlinear equations.

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Available software to do this?
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So we developed our own tool: ContactCenters, a Java library built over SSJ.

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Library based on well-supported modern programming language Java.
Strong expressive power. GUI-independent.
Interoperability with other software (statistics, optimization, databases, etc).

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Optimization: Integer Programming Formulation

Call types	$k = 1, \ldots, K$;
Agent types (or skill groups)	$i = 1, \ldots, I$;
Periods	$p = 1, \ldots, P$;
Shift types	$q = 1, \ldots, Q$.

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We have $\mathbf{y} = \mathbf{A}\mathbf{x}$ where $\mathbf{A}$ is block diagonal with i blocks $\tilde{\mathbf{A}}$, and element (p, q) of $\tilde{\mathbf{A}}$ is 1 if shift q covers period p, 0 otherwise.

$g_{k,p}(\mathbf{y})$ = long-run SL for call type k in period p.
E.g., fraction of calls answered within $\tau_{k,p}$ seconds:

$$g_{k,p}(\mathbf{y}) = \frac{E[\text{num. of calls in period } p \text{ answered within the limit}]}{E[\text{num. of arrivals in period } p]}.$$

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These functions $g_\bullet$ (ratios of expectations) are unknown and often very complicated. Each may depend on entire vector $\mathbf{y}$. They can be either

- approximated via simplified queueing models, or
- estimated by simulation.

Here, the routing rules are assumed fixed; we do not optimize them.
(But eventually we should.)

Scheduling problem

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Neglects the transient and overflow effect across periods.

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Potential discrepancy in mix of agent types across successive periods.
Can be alleviated by skill-transfer decision variables.

Skill-transfer decision variables [Bhulai, Koole, and Pot 2005]:
$z_{p,j,i}$ = number of agents of type j that work as type-i agents in period p (they use only part of their skills).

$$\min \quad \mathbf{c}^{\mathrm{t}}\mathbf{x} = \sum_{i=1}^{I}\sum_{q=1}^{Q} c_{i,q}x_{i,q}$$

subject to

$$\sum_{q:\tilde{A}_{p,q}=1} x_{i,q} + \sum_{j\in\mathcal{S}_i^+} z_{p,j,i} - \sum_{j\in\mathcal{S}_i^-} z_{p,i,j} \geq y_{i,p} \quad \forall p,\ i$$

all $x_{i,q}, z_{p,j,i} \geq 0$ and integer.

Sample-path optimization via simulation

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To compute them at different values of $\mathbf{y}$, we simply use simulation with well-synchronized common random numbers.

Empirical scheduling optimization problem (sample version of the problem):

$$\begin{array}{rll} \min & \mathbf{c}^{\mathrm{t}}\mathbf{x} = \sum_{i=1}^{I}\sum_{q=1}^{Q} c_{i,q}x_{i,q} & \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{y}, & \\ & G_{n,k,p}(\mathbf{y}) \geq l_{k,p} & \text{for all } k, p, \\ & G_{n,p}(\mathbf{y}) \geq l_{p} & \text{for all } p, \\ & G_{n,k}(\mathbf{y}) \geq l_{k} & \text{for all } k, \\ & G_{n}(\mathbf{y}) \geq l, & \\ & \mathbf{x} \geq 0, \text{ and integer.} & \end{array}$$

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Similar formulation for the staffing problem.

Linear / integer programming with cutting planes

[Atlason, Epelman, and Henderson, 2004; Cezik and L'Ecuyer 2005]

Start with a relaxation of the IP and add cuts (linear constraints) derived from the SL constraints that are not satisfied in the sample problem.
Stop when all SL constraints are satisfied.

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Initial constraints are obtained by imposing that skill supply covers the load.

A realistic staffing problem with 65 call types and 89 agent types was solved (approx.) by this approach.

Heuristic neighborhood search for staffing problem

[Avramidis et al. 2006]

Stage 1: Start with feasible solution and decrease cost using neighborhood search with “loss-delay” approximation to evaluate SL and select next solution.

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- the incumbent is infeasible and we seek a nearby feasible solution;
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This approach is competitive with the previous one, especially when the computing budget is small (according to our experiments).

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We have also assumed fixed routing rules, no retrials, etc.

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Stratify on most important variables (e.g., B).

Use control variates.

Variance reduction: examples

Suppose the arrival process is Poisson with rate $B\lambda(t)$ at time t, where $\mathbb{E}[B] = 1$ and $\lambda(\cdot)$ is deterministic.

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$X_i = G_i(s_0) =$ number of calls who waited less than s_0 seconds on day i. Crude estimator of $\mu = \mathbb{E}[X_i]$:

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Stratification on U in k strata: for $s = 1, \ldots, k$, generate n_s indep. r.v.'s $U^{(s)} \sim \mathrm{Unif}((s-1)/k,\ s/k)$ and denote $X_{s,1}, \ldots, X_{s,n_s}$ the corresponding observations of $G(s_0)$.

Stratified estimator:

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$$\mathrm{Var}[\bar{X}_{\mathrm{s},n}] = \frac{1}{k^2}\sum_{s=1}^{k}\sigma_s^2/n_s$$

where $\sigma_s^2 = \mathrm{Var}[X \mid S = s]$ (must be estimated), is minimized by taking (if we neglect rounding)

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We have

$$\mathrm{Var}[\bar{X}_n] \quad = \quad \mathrm{Var}[\bar{X}_{\mathrm{so},n}] + \frac{1}{nk}\sum_{s=1}^{k}(\sigma_s - \bar{\sigma})^2 + \frac{1}{nk}\sum_{s=1}^{k}(\mu_s - \mu)^2.$$

Combining with a control variate.

Let A = number of arrivals during the day. CV estimator conditional on $U = u$:

$$X_{\rm c}(u) = X - \beta(u)[A - \mathbb{E}[A \mid U = u]] = X - \beta(u)C(u).$$

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$$\beta^*(u) = \frac{\mathrm{Cov}[A, X|U = u]}{\mathrm{Var}[A|U = u]} = \frac{E[C \cdot X \mid U = u]}{E[C^2 \mid U = u]}.$$

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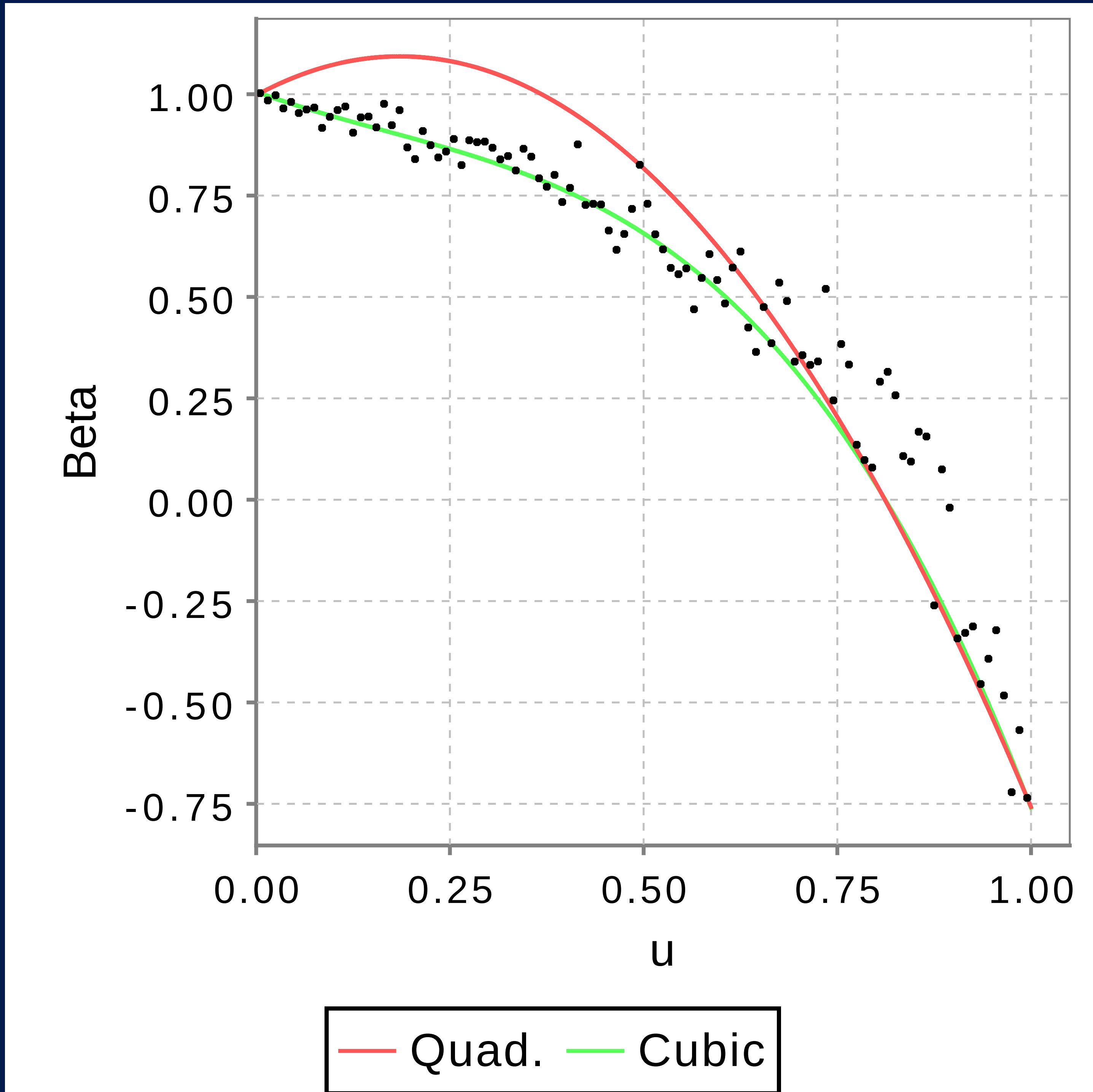
Variance in stratum s is now

$$\sigma_s^2 = \mathrm{Var}[X_{\rm c}(U^{(s)})] = \frac{1}{k}\int_{(s-1)/k}^{s/k} \mathrm{Var}[X - \beta^*(u)C \mid U = u]du$$

where $U^{(s)} \sim$ Unif($(s-1)/k,\ s/k$). So the optimal allocation changes!



The function $\beta^*(u)$ approximated by polynomial interpolation

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With well-synchronized CRN, we can prove that $\mathrm{Var}[\Delta_i] = O(\delta^{-1-\epsilon})$.

Ongoing and Future Work in Our Group

- Develop a Java library for simulation of call centers.
- Variance reduction methods for simulation.
- Integrate with optimization algorithms for staffing, scheduling, rostering, routing.
- Integrate with approximation formulas and CTMC models.
- Better models for inbound traffic and other aspects.
- Currently, our work is driven by the interests of Bell Canada.